

Deformation procedure for scalar fields in cosmology

D. Bazeia,^a L. Losano,^a and A.B. Pavan^b

^a*Departamento de Física, Universidade Federal da Paraíba
Caixa Postal 5008, 58051-970 João Pessoa PB, Brazil*

^b*Instituto de Física, Universidade de São Paulo
Caixa Postal 66318, 05315-970 São Paulo SP, Brazil*

(Dated: November 7, 2006)

This work offers an extension of the deformation procedure introduced in field theory to the case of standard cosmology in the presence of real scalar field in flat space-time. The procedure is shown to work for many models, which give rise to several different cosmic scenarios, evolving under the presence of first-order differential equations which solve the corresponding equations of motion very appropriately.

PACS numbers: 98.80.Cq; 04.20.Jb

I. INTRODUCTION

The recent discovery of cosmic acceleration [1, 2, 3] has brought to the stage one the main mysteries of modern cosmology. Named dark energy, it amounts to almost 70% of the total energy of the universe to cause the present cosmic acceleration. According to the recent understanding [4, 5, 6, 7, 8], one of the main fuel of the cosmic acceleration is quintessence [9, 10, 11, 12], which in standard Friedmann-Robertson-Walker (FRW) cosmology may be described by real scalar fields.

The presence of acceleration has raised a great deal of interest in cosmology driven by scalar fields, welcoming the construction of new cosmological models. Among the several possibilities, which are reviewed in Refs. [4, 5, 6, 7, 8], we are mainly interested in FRW cosmology driven by real scalar field with standard dynamics. In particular, the specific aim of the present work is inspired in the very interesting investigation concerning the accelerated expansion of the Universe done in Ref. [13].

Before going into the main issue, however, let us recall another line of investigation which develops alternative methodology. In the recent work [14] some of us have turned attention to FRW models described by real scalar fields, focusing on inspecting Einstein's equation and the equation of motion for the scalar field in a very direct way, considering models described by real scalar field in generic space-time, displaying the usual spheric, flat or hyperbolic spatial profile. The pioneer investigations show how to obtain first-order differential equations which solve the corresponding equations of motion, for cosmology in general spherical, flat or hyperbolic geometry. The presence of first-order equations was then extended to brane in [15], and this has also been explored in Refs. [16, 17, 18] in several contexts, including the case of brane with a single extra dimension, which has also appeared in the former investigations [19, 20, 21, 22, 23, 24, 25]. Evidently, the presence of first-order equations ease the process of solving specific models, finding analytical solutions which can be used to fully understand the related cosmic evolution.

The first-order formalism offers an interesting way to investigate FRW models in standard cosmology. In the present investigation our main motivation is to use the first-order formalism to extend the deformation procedure of Ref. [26] to modern cosmology. We believe that the possibility of deforming the cosmic evolution of some standard model will certainly lead to new possibilities of investigation, contributing to improve the present view of the subject. Indeed, in some recent works [27, 28] one has shown how to extend the deformation procedure of Ref. [26] to several other scenarios, including the Randall-Sundrum braneworld scenario [29] described by warped geometry with a single extra dimension of infinity extend, and these results may help us to further enlarge the deformation procedure to the case of braneworld with internal structure, as investigated for instance in Refs. [30, 31].

The key motivation for the present work has appeared after the work [13], which deals with FRW cosmology driven by real scalar field. The scalar field model is driven by some potential $V = V(\phi)$. In this case, if the scale factor $a(t)$ is known, the potential $V(t)$ and the field $\phi(t)$ are also known [13]. Thus, if one inverts $t = t(\phi)$ we can write $V = V(\phi)$. The lesson to learn here is that the cosmic evolution determines the potential. This is a very nice result, and we can use it together with the deformation procedure of Ref. [26], which introduces a direct approach to relate different models, described by different potentials. There we learn how to get from $V(\phi)$ to another potential, the deformed potential $\tilde{V}(\tilde{\phi})$. Thus, if we relate $V(\phi)$ to $a(t)$, then we can also relate $\tilde{V}(\tilde{\phi})$ to $\tilde{a}(t)$, and so we can relate $a(t)$ to $\tilde{a}(t)$. This is the deformation procedure, now extended to the cosmic evolution. As we show below, it works nicely for standard FRW cosmology in flat space-time, for models described by first-order differential equations.

To make the deformation procedure effective, for simplicity we deal with flat geometry, that is, we consider the line element

$$ds^2 = dt^2 - a^2(t)[dr^2 + r^2 d\Omega^2] \quad (1)$$

where $a(t)$ is the scale factor, and we also use $H = \dot{a}/a$ to describe the Hubble parameter. This is the standard

scenario, and now we include scalar field model with a single real scalar field. In this case, the Einstein-Hilbert action is given by

$$S = \int d^4x \sqrt{-g} \left(-\frac{1}{4} R + \mathcal{L}(\phi, \partial_\mu \phi) \right) \quad (2)$$

where ϕ describes the scalar field and we are using $4\pi G = 1$. In general, the energy-momentum tensor is given by $T^\mu_\nu = \text{diag}(\rho, -p, -p, -p)$, where ρ and p represent energy density and pressure. We use Einstein's equation to get

$$H^2 = \frac{2}{3} \rho \quad (3a)$$

$$\frac{\ddot{a}}{a} = -\frac{1}{3}(\rho + 3p) \quad (3b)$$

The equation of motion for the scalar field depend on $\mathcal{L}(\phi, \partial_\mu \phi)$, which has the standard form

$$\mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \quad (4)$$

The energy density and pressure that account for the scalar field model are given by

$$\rho = \frac{1}{2} \dot{\phi}^2 + V, \quad p = \frac{1}{2} \dot{\phi}^2 - V \quad (5)$$

and the equation of motion for the scalar field is given by

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0 \quad (6)$$

We use the energy density and pressure to rewrite Eqs. (3) in the form

$$\dot{H} = -\dot{\phi}^2, \quad (7a)$$

$$H^2 = \frac{1}{3} \dot{\phi}^2 + \frac{2}{3} V \quad (7b)$$

As one knows, the above Eqs. (6) and (7) constitute the equations that we have to solve in the case of FRW cosmology driven by scalar field with standard dynamics in flat space-time.

In this standard scenario in cosmology, the deformation procedure that we develop below may have connections with other interesting investigations. Some direct issues concern Refs. [32, 33]: in the first case, in Ref. [32] one investigates the generation of new solutions from existing solutions, taking advantage of the presence of invariances of Einstein's equations in FRW models described by a single real scalar field in flat space-time; in the second case, in Ref. [33] one deals with cosmological reconstruction of scalar field potential and dark energy. These investigations introduce new motivations and enlarge our interest in the subject.

II. FIRST-ORDER EQUATIONS

Before going to the subject we want to explore in the present work, let us firstly review the main steps of the first-order formalism put forward in Refs. [14, 17]. In FRW cosmology, it is the standard view that the scale factor, the scalar field and the Hubble parameter are all functions of the time evolution, that is, $a = a(t)$, $\phi = \phi(t)$, and $H = H(t)$. Also, from Einstein's equation we need to see the potential as a function of time too. However, from the equation of motion for the scalar field we have $V = V(\phi)$, which shows the potential as a function of the scalar field. Thus, to make these two views equivalent we can also think of the Hubble's parameter as a function of the scalar field. This is the key point, and we make it very efficient with the introduction of a new function – $W = W(\phi)$ – from which we can now understand that the Hubble parameter depends on time as a function of $W[\phi(t)]$.

A. The formalism

To make the above reasonings formal, we introduce $W = W(\phi)$ such that the mathematical expression which follows can be written as

$$H = W(\phi) \quad (8)$$

This is a first-order differential equation for the scale factor. It allows obtaining another equation, involving the scalar field, in the form

$$\dot{\phi} = -W_\phi \quad (9)$$

where W_ϕ stands for $dW/d\phi$. These Eqs. (8) and (9) imply that the potential has to have the form

$$V = \frac{3}{2} W^2 - \frac{1}{2} W_\phi^2 \quad (10)$$

This procedure in flat space-time is similar to the Hamilton-Jacobi approach of Ref. [36]. If instead of the deceleration parameter, we define $\bar{q} = \ddot{a}a/\dot{a}^2$ as the acceleration parameter, we get $\bar{q} = 1 + \dot{H}/H^2$. For the above model, the acceleration parameter has the form

$$\bar{q} = 1 - \frac{W_\phi^2}{W^2} \quad (11)$$

The energy density and pressure are given by, respectively

$$\rho(t) = \frac{3}{2} W^2, \quad p(t) = W_\phi^2 - \frac{3}{2} W^2 \quad (12)$$

and now we can write the equation of state $\omega = p/\rho$ in the form

$$\omega = \frac{2}{3} \frac{W_\phi^2}{W^2} - 1 \quad (13)$$

It is not constant, and from Eqs. (11) and (13) we see that for an accelerated expansion we must have $\omega < -1/3$.

B. An example

We illustrate the above results with some examples. Firstly, we take

$$W = A\phi^n + B \quad (14)$$

where A and B are real parameters. This leads to models defined by the potentials:

$$V(\phi) = \frac{3}{2} [A^2\phi^4 + 2AB\phi^2 + B^2] - 2A^2\phi^2 \quad (15)$$

for $n = 2$, and

$$V(\phi) = \frac{3}{2} [A^2\phi^{2n} + 2AB\phi^n + B^2] - \frac{A^2n^2\phi^{2n-2}}{2} \quad (16)$$

for $n \neq 2$. For these models, the set of Eqs. (6) and (7) are solved by

$$\phi(t) = e^{-2At}, \quad \text{for } n = 2 \quad (17)$$

$$\phi(t) = [An(n-2)t]^{\frac{1}{2-n}}, \quad \text{for } n \neq 2 \quad (18)$$

and the Hubble parameters

$$H(t) = Ae^{-4At} + B, \quad \text{for } n = 2 \quad (19)$$

$$H(t) = A[An(n-2)t]^{\frac{n}{2-n}} + B, \quad \text{for } n \neq 2 \quad (20)$$

These results allow obtaining the scale factors

$$a(t) = \exp\left(\frac{A(2-n)[An(n-2)]^{\frac{n}{2-n}}}{2} t^{\frac{2}{2-n}} + Bt\right) \quad (21)$$

for $n \neq 2$, and

$$a(t) = \exp\left(-\frac{1}{4}e^{-4At} + Bt\right), \quad \text{for } n = 2 \quad (22)$$

Here we need to pay attention to Eqs.(18), (20) and (21) when we choose $n \neq 2$, since it is necessary to adjust n and A to make all quantities real. We see that the scale factor in Eq. (21) leads to intermediate inflation, an issue set forward in Ref. [34].

In the above results, we consider the case for $n = 2$. In this case, A can be positive or negative. We first restrict our attention to the case A positive, and in Fig. 1 we plot both the scale factor a and Hubble's parameter H for some choice of parameters. Here we have three distinct sub-models: for $B = 0$ the universe expands, asymptotically approaching the static scenario; for $B > 0$ the universe expands eternally and for $B < 0$ the universe turns from an expanding phase to contraction. In late times, all the models behave according to e^{Bt} , that is, evolve to a de Sitter universe. For $A < 0$, the value of B does not qualitatively change the evolution, which is now similar to the former case, for $A > 0$ and $B < 0$.

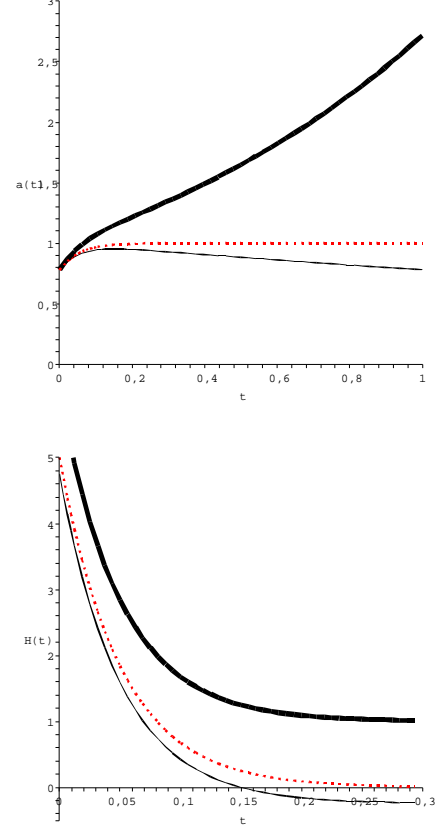


FIG. 1: The scale factor a (upper panel) and the Hubble's parameter H (lower panel) are plotted for $A = 5$, and $B = 0$ (dotted line), $B = 1$ (thick line), and $B = -1/4$ (thin line).

The energy density and pressure are given by

$$\rho(t) = \frac{3}{2} [Ae^{-4At} + B]^2 \quad (23)$$

$$p(t) = 4A^2e^{-4At} - \frac{3}{2} [Ae^{-4At} + B]^2 \quad (24)$$

In Fig. 2 we plot the energy density ρ and pressure p . Again, if we restrict our attention to the case $A > 0$, we can see that: for $B = 0$ the energy density and pressure are always positive, decreasing to zero asymptotically. For $B > 0$ the pressure is initially positive and it decreases asymptotically, going to a negative constant, while the energy density starts positive, decreases to its minimum ($\rho = 0$) and increases getting asymptotically to a positive constant. This case connects two distinct scenarios, with different matter contents. Finally, for $B < 0$, pressure and energy density behave similarly, as in the case with $B = 0$. If $A < 0$, the energy density increases, independently of the value of B , becoming exponentially infinity at later times; for the pressure, it decreases getting asymptotically to the negative infinity exponentially.

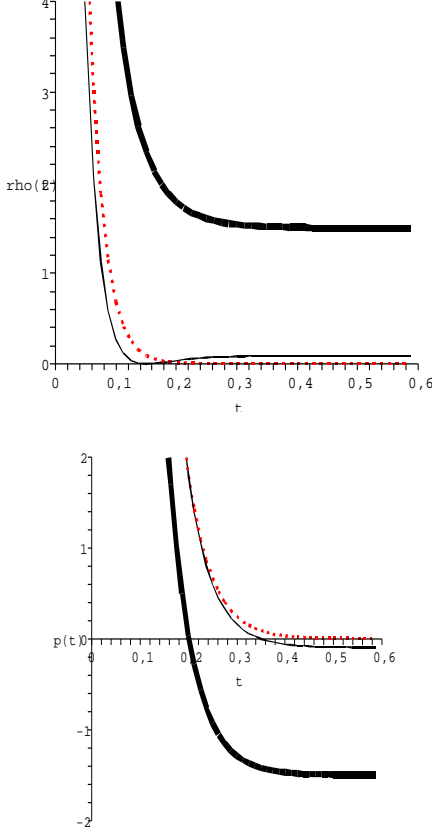


FIG. 2: The energy density ρ (upper panel) and the pressure p (lower panel) for $A = 5$, and $B = 0$ (dotted line), $B = 1$ (thick line), and $B = -1/4$ (thin line).

The acceleration parameter in this model is given by

$$\bar{q} = 1 - \left(\frac{2Ae^{-2At}}{B + Ae^{-4At}} \right)^2 \quad (25)$$

In Fig. 3 we plot the acceleration parameter \bar{q} and ω . In general, for $A > 0$ and with $B = 0$ the cosmic evolutions are always desaccelerated with $\omega > 0$; with $B > 0$ two phases are possible: one, decelerated, followed by another phase, accelerated, with ω starting in a positive value and ending in a negative one. In the last case, with $B < 0$, both \bar{q} and ω may diverge, when the energy density goes to zero and the pressure has a finite value.

C. Another example

We now introduce another example, described by

$$W = A \cosh(B\phi) \quad (26)$$

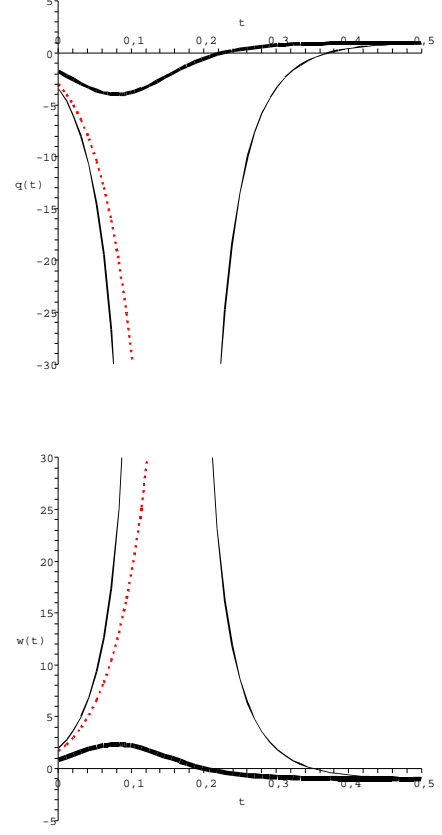


FIG. 3: The acceleration parameter $\bar{q}(t)$ (upper panel) and the ω parameter (lower panel) for $A = 5$, $B = 0$ (dotted line), $A = 5$, $B = 1$ (thick line) and $A = 5$, $B = -1/4$ (thin line).

where A and B are real parameters. This choice leads to the potential

$$V(\phi) = \frac{3}{2}A^2 \cosh^2(B\phi) - \frac{1}{2}A^2B^2 \sin^2(B\phi) \quad (27)$$

Similar potential was suggested to describe quintessence and dark matter in the work [35]. For the present potential, the set of Eqs. (6) and (7) are solved by

$$\phi(t) = \frac{1}{B} \ln \left(\tanh \left(\frac{AB^2t}{2} \right) \right) \quad (28)$$

and the Hubble's parameter has the form

$$H(t) = A \cosh \left(\ln \left(\tanh \left(\frac{AB^2t}{2} \right) \right) \right) \quad (29)$$

The scale factor becomes

$$a(t) = \left[\tanh \left(\frac{AB^2t}{2} \right)^2 - 1 \right]^{-1/B^2} \tanh \left(\frac{AB^2t}{2} \right)^{1/B^2} \quad (30)$$

We see from Eq. (28) that $A > 0$.

For this new case, we plot in Fig. 4 the scale factor and Hubble's parameter for some choice of parameters. The scale factor for $B^2 = 1/n$ with n even is always positive and increasing, and for n odd, it is negative, decreasing, and in both cases the evolution is accelerated, with the Hubble parameter positive, decreasing asymptotically to a constant value. This model appears to evolve from an initial singularity, somehow similar to the Big Bang model. In late times, we can see from the Hubble parameter that the cosmic evolution leads to de Sitter geometry.

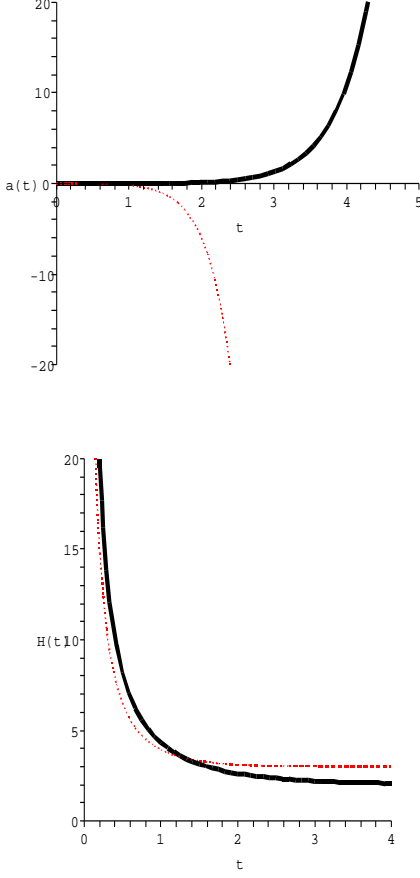


FIG. 4: Plots of the scale factor $a(t)$ (upper panel) and Hubble's parameter $H(t)$ (lower panel) for $A = 2$, $B^2 = 1/4$ (thick line) and $A = 3$, $B^2 = 1/3$ (dotted line).

In this new case the acceleration parameter has the form

$$\bar{q} = 1 - B^2 \tanh \left(\ln \left(\tanh \left(\frac{AB^2 t}{2} \right) \right) \right)^2 \quad (31)$$

and the energy density and pressure are given by

$$\rho(t) = \frac{3}{2} A^2 \cosh \left(\ln \left(\tanh \left(\frac{AB^2 t}{2} \right) \right) \right)^2 \quad (32)$$

$$p(t) = A^2 B^2 \sinh \left(\ln \left(\tanh \left(\frac{AB^2 t}{2} \right) \right) \right)^2 - \frac{3}{2} A^2 \cosh \left(\ln \left(\tanh \left(\frac{AB^2 t}{2} \right) \right) \right)^2 \quad (33)$$

In Fig. 5 we plot the acceleration and ω . We notice that ω varies in the interval $0 < \omega < -1$, going to -1 asymptotically, as it happens to be the case of FRW cosmology with cosmological constant [8].

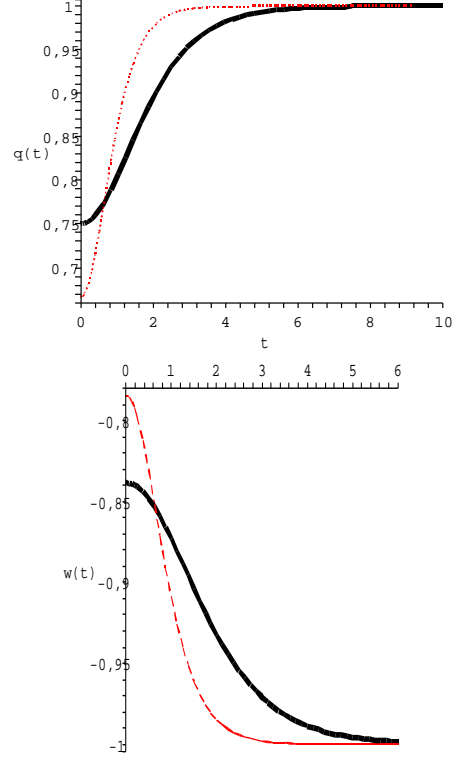


FIG. 5: The acceleration parameter $\bar{q}(t)$ (upper panel) and ω (lower panel) for $A = 2$, $B^2 = 1/4$ (thick line) and $A = 3$, $B^2 = 1/3$ (dashed line).

III. THE DEFORMATION PROCEDURE

We now turn attention to the deformation procedure introduced in Ref. [26]. The idea is to bring the methodology behind the deformation procedure to FRW cosmology, with the motivation that since the deformation change the original scalar field model, in the cosmological scenario it will also change the cosmic evolution. The approach may then be used to find families of soluble cosmological models, driven by a single real scalar field.

We follow the original work [26], but now we start with the Einstein-Hilbert action (2). We deform the scalar field using $\phi = f(\tilde{\phi})$, with f being the deformation func-

tion, a real function of the new field $\tilde{\phi}$. We suppose that $f : \mathbf{R} \rightarrow \mathbf{R}$ is bijective, thus invertible and differentiable.

For each model represented by the potential $V(\phi)$, giving by (10) and having solutions $\phi(t)$ and $H(t)$ satisfying Eqs. (8) and (9), and thus Eqs. (6) and (7), we can introduce another model, the deformed model, which is described by the deformed scalar field and is defined by

$$\tilde{V}(\tilde{\phi}) = \frac{3}{2}\tilde{W}^2 - \frac{1}{2}\tilde{W}_{\tilde{\phi}}^2 \quad (34)$$

with

$$\tilde{W} = \int_0^{\tilde{\phi}} \frac{W_{\phi}[f(\tilde{\phi})]}{df/d\tilde{\phi}} d\tilde{\phi} \quad (35)$$

The key point of the deformation procedure is that if $\phi(t)$ and $H(t) = W(\phi(t))$ solve the the original model, then the deformed model is solved by $\tilde{\phi}(t) = f^{-1}[\phi(t)]$ and $\tilde{H}(t) = \tilde{W}[\tilde{\phi}(t)]$, for f^{-1} being the inverse of f .

A. An example

We illustrate the above result with some examples. Firstly, we consider the model defined by Eq.(14) and the deformation function $f(\tilde{\phi}) = \ln \tilde{\phi}$. This leads to a new model described by the deformed potential

$$\tilde{V} = \frac{3n^2}{2} \left[\tilde{W}_1 \ln(\tilde{\phi})^{n-1} - \left(\frac{n-1}{2} \right) \tilde{W}_{n-1} \right]^2 + \frac{1}{2} A^2 n^2 \ln(\tilde{\phi})^{2n-2} \quad (36)$$

where

$$\tilde{W}_n = n \left[\tilde{W}_1 \ln(\tilde{\phi})^{n-1} - \frac{n-1}{2} \tilde{W}_{n-1} \right] + C_n \quad (37)$$

for $n \geq 2$, and for $n = 1$

$$\tilde{W}_1 = \frac{1}{2} A \tilde{\phi}^2 \quad (38)$$

For this model, from Eq.(17) the deformed field is obtained as $\tilde{\phi} = \exp(\phi)$; thus, we can use Eq. (17) to write

$$\tilde{\phi}_n(t) = e^{[An(n-2)t]^{\frac{1}{2-n}}} \quad n \neq 2 \quad (39)$$

$$\tilde{\phi}_2(t) = e^{e^{-2At}} \quad n = 2 \quad (40)$$

The Hubble parameter has the following form

$$\tilde{H}_n(t) = \frac{1}{2} n A e^{2[An(n-2)t]^{\frac{1}{2-n}}} [An(n-2)t]^{\frac{n-1}{2-n}} + \frac{1}{2} n(n-1) \tilde{W}_{n-1} + C_n \quad n \neq 2 \quad (41)$$

and

$$\tilde{H}_2(t) = A e^{2e^{-2At}} \left(e^{-2At} - \frac{1}{2} \right) + C_2 \quad n = 2 \quad (42)$$

and the acceleration parameter gets to

$$\tilde{q}_2(t) = 1 - \left(\frac{2A\tilde{\phi} \ln(\tilde{\phi})}{A\tilde{\phi}^2(\ln(\tilde{\phi}) - 1) + C_2} \right)^2 \quad \text{for } n = 2 \quad (43)$$

and

$$\tilde{q}_n(t) = 1 - n^2 \frac{N_1^2}{N_2^2} \quad \text{for } n \neq 2 \quad (44)$$

where we have set

$$N_1 = 2\tilde{\phi} A \ln(\tilde{\phi})^{(n-1)} + (n-1) A \tilde{\phi} \ln(\tilde{\phi})^{(n-2)} + (n-1) \frac{d\tilde{W}_{n-1}}{d\tilde{\phi}} \quad (45)$$

and

$$N_2 = n A \tilde{\phi}^2 \ln(\tilde{\phi})^{(n-1)} - n(n-1) \tilde{W}_{n-1} + 2 C_n \quad (46)$$

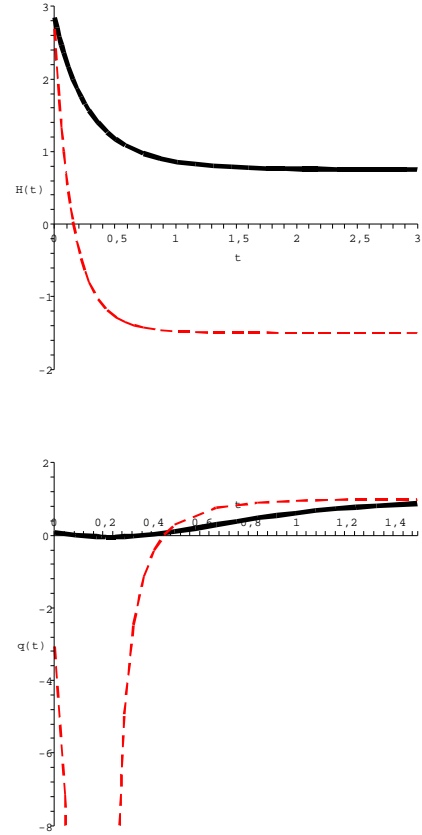


FIG. 6: Plots of the Hubble's parameter $\tilde{H}(t)$ (upper panel) and the acceleration parameter $\tilde{q}(t)$ (lower panel) for $A = 1$, $C_2 = -1$ (dashed line), and for $A = 1/2$, $C_2 = 1$ (thick line).

In this case, we could not obtain analytic expressions for the scale factor and so we use \tilde{H} and \tilde{q} to analyze the above cosmic evolution; they are plotted in Fig. (6). We see from Fig. (6) that the constant C_2 can be used to make $\tilde{q}(t)$ divergent.

B. Another example

Here we introduce another example, described by the function $W = A \cosh(B\phi) + D$. And also, we use the deformation function $f(\tilde{\phi}) = \frac{1}{B} \ln(\tanh(E \sinh(B\tilde{\phi})))$. A , B , D , and E are real parameters. In this case the new model is represented by the deformed potential

$$\tilde{V} = \frac{3}{2} \left(D - \frac{A}{E} \arctan(e^{B\tilde{\phi}}) \right)^2 - \frac{1}{8} \frac{A^2 B^2}{E^2 \cosh^2(B\tilde{\phi})} \quad (47)$$

and from the inverse of the deformation function we have

$$\tilde{\phi} = \frac{1}{B} \operatorname{arcsinh} \left(\frac{AB^2 t}{2E} \right) \quad (48)$$

The deformed Hubble parameter is given by

$$\tilde{H}(t) = -\frac{A}{E} \arctan \left(\frac{AB^2 t + E \sqrt{4E^2 + A^2 B^4 t^2}}{2E} \right) + D \quad (49)$$

and the deformed scale factor is

$$\tilde{a}(t) = N_3^{\frac{1}{B^2}} \exp \left(-\frac{At}{E} \arctan \left(\frac{AB^2 t + N_3}{2E} \right) + Dt \right) \quad (50)$$

where we have set

$$N_3 = (4E^2 + A^2 B^4 t^2)^{1/2} \quad (51)$$

In Fig. 7 we plot the scale factor $\tilde{a}(t)$ and the Hubble parameter $\tilde{H}(t)$, for some choice of parameters, with $D \neq 0$. We see that the behavior of the scale factor follows three generic possibilities: increasing, increasing-decreasing, and decreasing. The Hubble parameter, however, decreasing in these cases, getting to a positive or negative constant asymptotically. This model has an interesting feature, related to the many distinct scenarios available from the many specific choice of parameters. We notice, however, that there is no cosmic evolution with initial singularity, as it happens with the Big Bang model.

In the present case, the acceleration parameter gets to the form

$$\tilde{q} = 1 - \frac{A^2 B^2}{4E^2} \left[N_4 \cosh \left(\operatorname{arcsinh} \left(\frac{AB^2 t}{2E} \right) \right) \right]^{-2} \quad (52)$$

where we have set

$$N_4 = -\frac{A}{E} \arctan \left(e^{\operatorname{arcsinh} \left(\frac{AB^2 t}{2E} \right)} \right) + D \quad (53)$$

Also, the energy density and pressure are given by

$$\tilde{\rho}(t) = \frac{3}{2} \left(D - \frac{A}{E} \arctan \left(e^{\operatorname{arcsinh} \left(\frac{AB^2 t}{2E} \right)} \right) \right)^2 \quad (54)$$

$$\tilde{p}(t) = \frac{1}{4} \frac{A^2 B^2}{E^2 \cosh^2 \left(\operatorname{arcsinh} \left(\frac{AB^2 t}{2E} \right) \right)} - \tilde{\rho}(t) \quad (55)$$

These quantities can also be examined similarly.

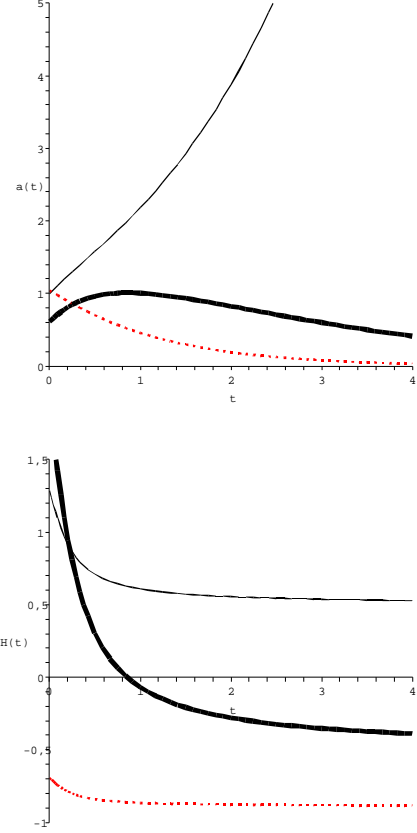


FIG. 7: The scale factor $\tilde{a}(t)$ (upper panel) and the Hubble's parameter $\tilde{H}(t)$ (lower panel) for $A = -1/2$, $B = 3$, $D = 1/2$, $E = 1/2$ (thin line), $A = -1/2$, $B = 3/2$, $D = -1/2$, $E = 1/6$ (thick line), $A = 1/2$, $B = 6$, $D = -1/2$, $E = 2$ (dotted line).

IV. ENDING COMMENTS

In this work we have revised and included new examples of the first-order formalism put forward in Ref. [14]. We did this in the simpler case of flat space-time, to provide the cosmological scenario needed to implement our key idea, of bringing to the cosmological scenario the methodology behind the deformation procedure put forward in Ref. [26] in the absence of gravity.

The main results of the present work show that the deformation procedure can be nicely implemented for FRW models driven by scalar field under the first-order formalism, which helps significantly the search for analytical solutions. As we have shown, the deformation procedure introduced in this work may lead to new and interesting scenarios, obtained from simpler models of FRW cosmology. In general, we can deform an expanding model into another, expanding or contracting model. These possibilities depend not only on the deformation function, but also on the parameters which control the model to be deformed.

The present methodology may be used to investigate several interesting issues, in particular the case in which the cosmic evolution occurs in closed, flat or open geometry, for scalar field evolving under standard or tachyonic dynamics. Another issue concerns the use of the deformation procedure for models in which the equations of motion cannot be reduced to first-order equations. The investigations will certainly enlighten the relation between the deformation procedure and the approaches introduced in Refs. [32, 33]. These extensions and other related issues are presently under consideration, and we

hope to report on them in the near future.

Acknowledgments

We would like to thank CNPq, FAPESP, PADCT/CNPq and PRONEX/CNPq/FAPESP for financial support. ABP thanks E. Abdalla for helping him to visit Departamento de Física, Universidade Federal da Paraíba.

-
- [1] A. Riess et al., *Astron. J.* **116**, 1009 (1998).
 - [2] S.J. Perlmutter et al., *Astrophys. J.* **517**, 565 (1999).
 - [3] N.A. Bachall, J.P. Ostriker, S.J. Perlmutter, and P.J. Steinhardt, *Science*, **284**, 1481 (1999).
 - [4] P.J.E. Peebles and B. Ratra, *Rev. Mod. Phys.* **75**, 559 (2003).
 - [5] T. Padmanabham, *Phys. Rep.* **380**, 235 (2003).
 - [6] M. Trodden and S.M. Carroll, *TASI lectures: introduction to cosmology*, astro-ph/0401547.
 - [7] A. Linde, *Inflation and string cosmology*, hep-th/0503195.
 - [8] E.J. Copeland, M. Sami, and S. Tsujikawa, *Dynamics of dark energy*, hep-th/0603057.
 - [9] J. Frieman, C. Hill, A. Stebbins, and I. Waga, *Phys. Rev. Lett.* **75**, 2077 (1995).
 - [10] R.R. Caldwell, R. Dave, and P.J. Steinhardt, *Phys. Rev. Lett.* **80**, 1582 (1998).
 - [11] S.M. Carroll, *Phys. Rev. Lett.* **81**, 3067 (1998).
 - [12] I. Zlatev, L. Wang, and P.J. Steinhardt, *Phys. Rev. Lett.* **82**, 896 (1999).
 - [13] T. Padmanabham, *Phys. Rev. D* **66**, 021301(R) (2002).
 - [14] D. Bazeia, C.B. Gomes, L. Losano, and R. Menezes, *Phys. Lett. B* **633**, 415 (2006).
 - [15] V.I. Afonso, D. Bazeia, and L. Losano, *Phys. Lett. B* **634**, 526 (2006).
 - [16] K. Skenderis and P.K. Townsend, *Phys. Rev. Lett.* **96**, 191301 (2006).
 - [17] D. Bazeia, L. Losano, and J.J. Rodrigues, *First-order formalism for scalar field in cosmology*, hep-th/0610028.
 - [18] K. Skenderis and P.K. Townsend, *Hamilton-Jacobi for domain walls and cosmologies*, hep-th/0609056.
 - [19] K. Skenderis and P.K. Townsend, *Phys. Lett. B* **468**, 46 (1999).
 - [20] O. DeWolfe, D.Z. Freedman, S.S. Gubser, and A. Karch, *Phys. Rev. D* **62**, 046008 (2000).
 - [21] F.A. Brito, M. Cvetič, and S.-C. Yoon, *Phys. Rev. D* **64**, 064021 (2001).
 - [22] M. Cvetič and N.D. Lambert, *Phys. Lett. B* **540**, 301 (2002).
 - [23] D. Bazeia, F.A. Brito, and J.R. Nascimento, *Phys. Rev. D* **68**, 085007 (2003).
 - [24] D.Z. Freedman, C. Núñez, M. Schnabl, and K. Skenderis, *Phys. Rev. D* **69**, 104027 (2004).
 - [25] A. Celi et al. *Phys. Rev. D* **71**, 045009 (2005).
 - [26] D. Bazeia, L. Losano, and J.M.C. Malbouisson, *Phys. Rev. D* **66**, 101701(R) (2002).
 - [27] C.A. Almeida, D. Bazeia, L. Losano, and J.M.C. Malbouisson, *Phys. Rev. D* **69**, 067702 (2004); D. Bazeia and L. Losano, *Phys. Rev. D* **73**, 025016 (2006); D. Bazeia, M.A. Gonzalez Leon, L. Losano, and J. Mateos Guilarte, *Phys. Rev. D* **73**, 105008 (2006); V.I. Afonso, D. Bazeia, and F.A. Brito, *JHEP* **08**, 073 (2006).
 - [28] A. de Souza Dutra, *Phys. Lett. B* **626**, 249 (2005); A. de Souza Dutra and A.C. Amaro de Faria, *Phys. Rev. D* **72**, 087701 (2005).
 - [29] L. Randall and R. Sundrum, *Phys. Rev. Lett.* **83**, 4690 (1999).
 - [30] D. Bazeia, J. Menezes, and R. Menezes, *Phys. Rev. Lett.* **91**, 241601 (2003); D. Bazeia, C. Furtado, and A.R. Gomes, *JCAP* **02**, 002 (2004); D. Bazeia and A.R. Gomes, *JHEP* **05**, 012 (2004).
 - [31] A. Melfo, N. Pantoja and A. Skrzewski, *Phys. Rev. D* **67**, 105003 (2003); O. Castillo-Felisola, A. Melfo, N. Pantoja, and A. Ramírez, *Phys. Rev. D* **70**, 104029 (2004).
 - [32] P. Parsons and J.D. Barrow, *Class. Quantum Grav.* **12**, 1715 (1995).
 - [33] T.D. Saini, S. Raychaudhury, V. Sahni, and A.A. Starobinsky, *Phys. Rev. Lett.* **85**, 1162 (2000); V. Sahni and A.A. Starobinsky, *Reconstructing dark energy*, astro-ph/0610026.
 - [34] J.D. Barrow, *Phys. Lett. B* **235**, 40 (1990); J.D. Barrow and P. Saich, *Phys. Lett. B* **249**, 406 (1990); J.D. Barrow and A.R. Liddle, *Phys. Rev. D* **47**, 5219(R) (1995).
 - [35] V. Sahni and L.-M. Wang, *Phys. Rev. D* **62**, 103517 (2000).
 - [36] A.R. Liddle and D.H. Lyth, *Cosmological inflation and large-scale structure* (Cambridge, NY, 2000).